Find the first 3 terms, in ascending powers of $\mathbf{x}$, of the binomial expansion of $(3-2\mathbf{x})^5$, giving each term in its simplest form.

(4)

(a) Find the first 4 terms, in ascending powers of $\mathbf{x}$, of the binomial expansion of $(1+\mathbf{ax})^7$, where $\mathbf{a}$ is a constant. Give each term in its simplest form.

(4)

Given that the coefficient of $\mathbf{x}^2$ in this expansion is 525,

(b) find the possible values of $\mathbf{a}$.

(2)

Binomial Expansion

(a) Find the first 4 terms of the binomial expansion, in ascending powers of x, of

$$\left(1+\frac{x}{4}\right)^8$$

giving each term in its simplest form.

(4)

(b) Use your expansion to estimate the value of $(1.025)^8$, giving your answer to 4 decimal places.

(3)

(a) Find the first four terms, in ascending powers of x, in the binomial expansion of $(1+kx)^6$, where k is a non-zero constant.

(3)

Given that, in this expansion, the coefficients of x and x^2 are equal, find

(b) the value of k,

(2)

(c) the coefficient of x^3.

(1)

Binomial Expansion

$(3-2x)^5 \approx (3)^5 + \binom{5}{1}(3)^4(-2x)^1 + \binom{5}{2}(3)^3(-2x)^2 + \ldots$

$\approx 243 - 810x + 1080x^2$

$(1+ax)^7 \approx (1)^7 + \binom{7}{1}(1)^6(ax) + \binom{7}{2}(1)^5(ax)^2$
$\qquad\qquad\qquad + \binom{7}{3}(1)^4(ax)^3$

$(1+ax)^7 \approx 1 + 7ax + 21a^2x^2 + 35a^3x^3$

$21a^2 = 525$

$a^2 = \dfrac{525}{21} = 25$

$a = \pm\sqrt{25} = 5, -5$

$\boxed{a = 5}$
$\boxed{a = -5}$

$\left(1+\dfrac{x}{4}\right)^8 = 1 + 2x + \dfrac{8(7)}{2}\left(\dfrac{x}{4}\right)^2 + \dfrac{8(7)(6)}{6}\left(\dfrac{x}{4}\right)^3$

$\begin{bmatrix} n=8 \\ x = \frac{x}{4} \end{bmatrix} = 1 + 2x + \dfrac{7}{4}x^2 + \dfrac{7}{8}x^3$

$1 + \dfrac{x}{4} = 1.025 \qquad \therefore \dfrac{x}{4} = 0.025 \qquad \therefore x = 0.1$

using $x = 0.1$: $(1.025)^8 \approx 1 + 2(0.1) + \dfrac{7}{4}(0.1)^2 + \dfrac{7}{8}(0.1)^3$

$\approx \boxed{1.2184}$

$(1+kx)^6 = 1 + 6kx + \dfrac{6(6-1)}{2}(kx)^2 + \dfrac{6(6-1)(6-2)}{3!}(kx)^3$

$= 1 + 6kx + 15k^2x^2 + 20k^3x^3$

coeffs of $x = x^2$

$6k = 15k^2$
$6 = 15k$
$k = \dfrac{6}{15} = \dfrac{2}{5}$

coeff of $x^3 = 20k^3$