Mark scheme

Question 1

$y = -x + 6$

Parallel lines have same gradients, so the gradient of the line is $-$

The equation can be written in the form $y = mx + c$ where m is the gradient and c is the y-intercept.

Substituting the gradient gives $y = -x + c$

Substituting $x = 5$ and $y = 1$ give $1 = - \times\ 5 + c$

Rearranging gives $c = 1\ +\ 1 \times\ 5 = 6$

Therefore the equation is $y = -x + 6$

Question 2

$y = x + 1$

Parallel lines have same gradients, so the gradient of the line is

The equation can be written in the form $y = mx + c$ where m is the gradient and c is the y-intercept.

Substituting the gradient gives $y = x + c$

Substituting $x = 3$ and $y = 4$ give $4 =\times\ 3 + c$

Rearranging gives $c = 4\ -\ 1 \times\ 3 = 1$

Therefore the equation is $y = x + 1$

Question 3

$y = -\frac{1}{3}x + 4$

① Find the gradient of the line

$m = 3$

② Find the negative reciprocal of $m = 3$

$m_\perp\ =\ -\frac{1}{3}$

③ Substitute $m = -\frac{1}{3}$, $x = 3$ and $y = 3$ into $y = mx + c$

$$3 = -\frac{1}{3} \times 3 + c$$
$$c = 3 + \frac{1}{3} \times 3$$
$$= 4$$

④ Substitute $m = -\frac{1}{3}$ and $c = 4$ into $y = mx + c$

$y = -\frac{1}{3}x + 4$

Question 4

$y = -\frac{1}{2}x + 6$

① Find the gradient of the line

$m = 2$

② Find the negative reciprocal of $m = 2$

$m_{\perp} = -\frac{1}{2}$

③ Substitute $m = -\frac{1}{2}$, $x = 2$ and $y = 5$ into $y = mx + c$

$$5 = -\frac{1}{2} \times 2 + c$$
$$c = 5 + \frac{1}{2} \times 2$$
$$= 6$$

④ Substitute $m = -\frac{1}{2}$ and $c = 6$ into $y = mx + c$

$y = -\frac{1}{2}x + 6$

Question 5

$y - 1 = -2\left(x - \frac{9}{2}\right)$

Substitute $m = -2$, $x_1 = \frac{9}{2}$ and $y_1 = 1$ into $y - y_1 = m(x - x_1)$

$y - 1 = -2\left(x - \frac{9}{2}\right)$

Question 6

$y - 18 = -\frac{5}{4}(x - 10)$

Substitute $m = -\frac{5}{4}$, $x_1 = 10$ and $y_1 = 18$ into $y - y_1 = m(x - x_1)$

$y - 18 = -\frac{5}{4}(x - 10)$

Question 7

$y + 8 = -2(x + 7)$

Make y the subject of $-8x - 4y + 4 = 0$

$$\begin{aligned} -8x - 4y + 4 &= 0 \\ -8x + 4 &= 4y \\ -2x + 1 &= y \end{aligned}$$

Parallel lines have same gradient therefore $m = -2$

Substitute $m = -2$, $x_1 = -7$ and $y_1 = -8$ into $y - y_1 = m(x - x_1)$

$y + 8 = -2(x + 7)$

Question 8

$y + \frac{11}{2} = 2(x - 5)$

Make y the subject of $8x - 4y - 8 = 0$

$$\begin{aligned} 8x - 4y - 8 &= 0 \\ 8x - 8 &= 4y \\ 2x - 2 &= y \end{aligned}$$

Parallel lines have same gradient therefore $m = 2$

Substitute $m = 2$, $x_1 = 5$ and $y_1 = -\frac{11}{2}$ into $y - y_1 = m(x - x_1)$

$y + \frac{11}{2} = 2(x - 5)$

Question 9

$y + 13 = -\frac{4}{7}(x - 17)$

The gradients of perpendicular lines satisfy $m_{\perp} = -\frac{1}{m}$.

Given that $m = \frac{7}{4}$, $m_{\perp} = -\frac{4}{7}$

Substitute $m_{\perp} = -\frac{4}{7}$, $x_1 = 17$ and $y_1 = -13$ into $y - y_1 = m(x - x_1)$

$y+13=-\frac{4}{7}(x-17)$

Question 10

$y-11=(x+6)$

Make y the subject of $-2x-2y+10=0$

$$-2x-2y+10=0$$
$$-2x+10=2y$$
$$-x+5=y$$

The gradients of perpendicular lines satisfy $m_\perp = -\frac{1}{m}$.

Given that $m=-1$, $m_\perp=1$

Substitute $m_\perp=1$, $x_1=-6$ and $y_1=11$ into $y-y_1 = m(x-x_1)$

$y-11=(x+6)$

Question 11

$y=\frac{1}{3}x-1$

① Find the midpoint of PQ

$$\text{Midpoint}=\left(\frac{-2+2}{2},\frac{5+(-7)}{2}\right)$$

$$\text{Midpoint}=(0,-1)$$

② Find the perpendicular gradient.

$$m_{PQ}=\frac{-7-5}{2-(-2)}$$

$$m_{PQ}=-3$$

$$m_\perp = -\frac{1}{m_{PQ}}$$

$$\therefore\ m_\perp=\frac{1}{3}$$

③ Substitute into straight line equation.

$$y+1=\frac{1}{3}x$$
$$y+1=\frac{1}{3}x$$
$$y=\frac{1}{3}x-1$$

Question 12

$3x - y - 4 = 0$

① Find the midpoint of PQ

$$\text{Midpoint} = \left(\frac{7+1}{2}, \frac{7+9}{2}\right)$$

$$\text{Midpoint} = (4,8)$$

② Find the perpendicular gradient.

$$m_{PQ} = \frac{9-7}{1-7}$$

$$m_{PQ} = -\frac{1}{3}$$

$$m_{\perp} = -\frac{1}{m_{PQ}}$$

$$\therefore \ m_{\perp} = 3$$

③ Substitute into straight line equation.

$$y - 8 = 3(x - 4)$$
$$y - 8 = 3x - 12$$
$$3x - y - 4 = 0$$

Question 13

$\frac{25}{9}$

Coordinates of A by solving simultaneously $y = 4x$ and $2y + x = 10$:

$$2(4x) + x = 10$$
$$x = \frac{10}{9}$$

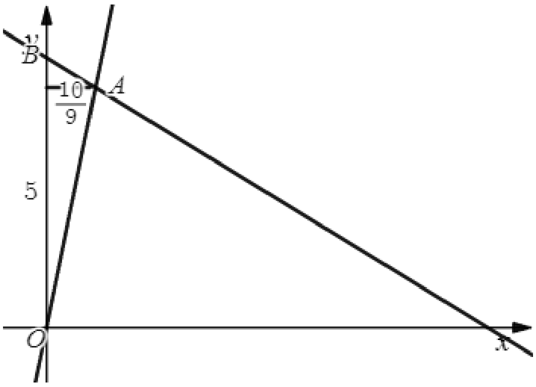
$$y = 4\left(\frac{10}{9}\right)$$
$$y = \frac{40}{9}$$

$$\therefore \ A\left(\frac{10}{9}, \frac{40}{9}\right)$$

Coordinates of B where $x = 0$:

$$y = -\frac{1}{2}(0) + 5$$
$$y = 5$$

$$\therefore \ B\ (0, 5)$$



Area of OAB:

$\frac{1}{2} \times 5 \times \frac{10}{9} = \frac{25}{9}$

Question 14

3

Coordinates of P by solving simultaneously $y = 4x$ and $y + 2x = 6$:

$$(4x) + 2x = 6$$
$$x = 1$$

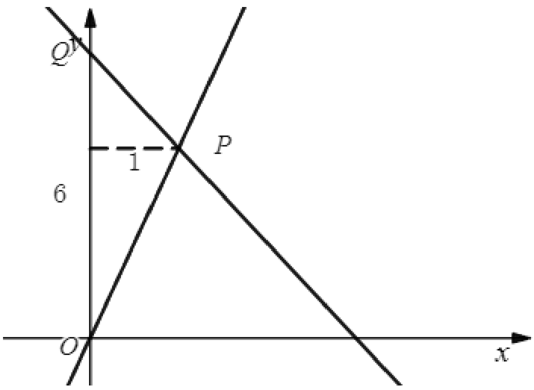
$$y = 4(1)$$
$$y = 4$$

$\therefore\ P\ (1, 4\ )$

Coordinates of Q where $x = 0$:

$$y = -2(0) + 6$$
$$y = 6$$

$\therefore\ Q\ (0, 6\ )$



Area of OPQ:

$\frac{1}{2} \times 6 \times 1 = 3$

Question 15

$\frac{36}{5}$

Coordinates of P by solving simultaneously $y = 2x$ and $2y + x = 12$:

$$2(2x) + x = 12$$
$$x = \frac{12}{5}$$

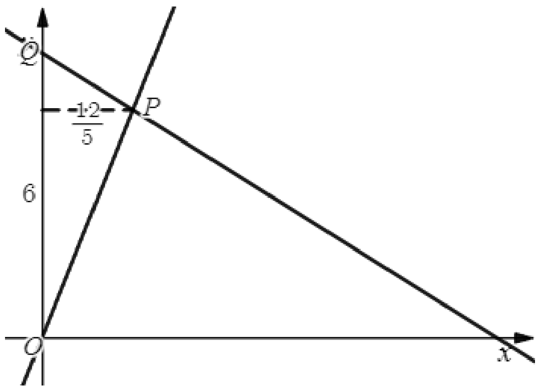
$$y = 2\left(\frac{12}{5}\right)$$
$$y = \frac{24}{5}$$

$\therefore\ P\left(\frac{12}{5}, \frac{24}{5}\right)$

Coordinates of Q where $x = 0$:

$$y = -\frac{1}{2}(0) + 6$$
$$y = 6$$

$\therefore\ Q\ (0, 6\ )$



Area of OPQ:

$\frac{1}{2} \times 6 \times \frac{12}{5} = \frac{36}{5}$

Question 16

$9 < h(x) < 19$

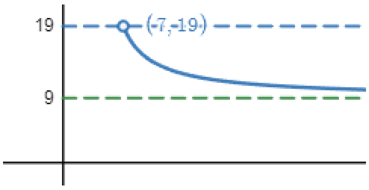
① Substitute $x = 7$ into $h(x)$ to find an initial y coordinate.

$$h(7) = \frac{18 \times 7 + 7}{2 \times 7 - 7}$$
$$= \frac{133}{7}$$
$$= 19$$

② Consider the limit of $h(x)$ as x increases.

$$\begin{aligned}\lim_{x\to\infty} h(x) &= \lim_{x\to\infty} \frac{18x+7}{2x-7} \\ &= \lim_{x\to\infty} \frac{18+\frac{7}{x}}{2-\frac{7}{x}} \\ &= \frac{18}{2} \\ &= 9\end{aligned}$$

③ Sketch a graph of $y = h(x)$ to visualise the function.



④ Use the graph to identify the range of $h(x)$.

[m]\begin{aligned}9

Question 17

$(4, 5)$

Complete the square:

$$\begin{aligned}y &= x^2 - 8x + 21 \\ &= (x-4)^2 - 16 + 21 \\ &= (x-4)^2 + 5\end{aligned}$$

Therefore the minimum point is $(4, 5)$

Question 18

$4p + 6$

You need to replace 25 by 5^2, and then multiply the powers.

$$\begin{aligned}25^{2p+3} &= \left(5^2\right)^{2p+3} \\ &= 5^{4p+6}\end{aligned}$$

$\therefore\ q = 4p + 6$

Question 19

$x = 5, y = -2$ or $x = -\frac{23}{5}, y = \frac{14}{5}$

Rearrange $x + 2y = 1$ to make y the subject.

$$x + 2y = 1$$
$$2y = 1 - x$$
$$y = \frac{1-x}{2}$$

Substitute $y = \frac{1-x}{2}$ into $x^2 + y^2 = 29$ then solve.

$$x^2 + \left(\frac{1-x}{2}\right)^2 = 29$$
$$x^2 + \frac{(1-x)(1-x)}{4} = 29$$
$$4x^2 + (1-x)(1-x) = 116$$
$$4x^2 + 1 - x - x + x^2 = 116$$
$$5x^2 - 2x + 1 = 116$$
$$5x^2 - 2x - 115 = 0$$
$$\therefore\ x = 5 \text{ or } x = -\frac{23}{5}$$

Substitute these values into $x + 2y = 1$

$$\text{When } x = 5,\ 1(5) + 2y = 1$$
$$y = -2$$

$$\text{When } x = -\frac{23}{5},\ 1\left(-\frac{23}{5}\right) + 2y = 1$$
$$y = \frac{14}{5}$$

Question 20

$-(y-2)^2 + 12$

① Factorise the value of -1 out of the first two terms.

$$-y^2 + 4y + 8 = -[y^2 - 4y] + 8$$

② Complete the square on the first two terms.

$$-[y^2 - 4y] + 8 = -[(y-2)^2 - 4] + 8$$

③ Expand the square brackets and simplify.

$$-[(y-2)^2 - 4] + 8$$
$$= -(y-2)^2 + 4 + 8$$

$$= -(y-2)^2 + 12$$

Year 12 AS Level Maths Homework Key Skills 11: Due Date: _______ Name: _____________________ Score: /20

Answer all questions, <u>make sure you show all working otherwise you will not be awarded marks</u>. If you write on any other paper, please hand this in with the sheet.

1. Find an equation of the line parallel to y=-x-2 and that passes through the point (5,1)

2. Find an equation of the line parallel to y = x* 5 and that passes through the point (5,1)

3. Find an equation of the line perpendicular to y = 3x - 4 and passes through the point (3,3)

4. Find an equation of the line perpendicular to y = 2x + 1 and passes through the point (2,5)

5. Find an equation of the line with the gradient -2 and passes through the point (9/2, 1)

6. Find an equation of the line with the gradient -5/4 and passses through the point (10,18)

7. Determine an equation of the line parallel to -8x - 4y + 4 = 0 and that passes through the point (-7,-8)

8. Determine an equation of the line parallel to 8x - 4y - 8 = 0 and that passes through the point (5, -11/2)

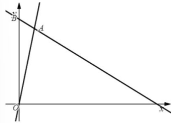
9. Find an equation of the line perpendicular to y= 7/4x - 3/2 and passes through the point (17, -13)

10. Find an equation of the line perpendicular to -2x -2y +10 =0 and passes through the point (-6,11)

11. A straight line passes through the points P(2, -7) and Q(2, -7) Find the equation of the perpendicular bisector of PQ. Give your answer in the form y=mx + c. Simplify your answer where possible.

12. A straight line passes through the points P(7, 7) and Q(1,9) Find the equation of the perpendicular bisector of PQ. Give your answer in the form ax + by + c = 0, where a,b and c are integers..

13. The line l_1 has the equation y = 4x. The line l_2 has the equation 2y + x = 10. The lines intersect at A. The line l_2 intersects the y axis at B. Find the exact area of triangle OAB



14. The line l_1 has the equation y = 4x. The line l_2 has the equation y +2 x = 6. The lines intersect at P. The line l_2 intersects the y axis at Q. Find the exact area of triangle OPQ.
HINT: Draw a diagram

15. The line l_1 has the equation y = 2x. The line l_2 has the equation 2y + x = 12. The lines intersect at P. The line l_2 intersects the y axis at Q. Find the exact area of triangle OPQ.
HINT: Draw a diagram

16. State the range of h.

$$h(x) = \frac{18x+7}{2x-7}\ ,\ x > 7$$

17. Find the minimum point of the graph

$$y = x^2 - 8x + 21$$

18. Express 25^{2p+3} in the form 5^q, stating q in terms of p

19. Solve the simultaneous equations

$$x + 2y = 1$$
$$x^2 + y^2 = 29$$

20. Write $-y^2 + 4y + 8$ in the form $a(y+b)^2 + c$. Where a, b and c are rational numbers.