The first term of a geometric series is 120. The sum to infinity of the series is 480.

(a) Show that the common ratio, r, is $\frac{3}{4}$.

(3)

(b) Find, to 2 decimal places, the difference between the 5th and 6th term.

(2)

(c) Calculate the sum of the first 7 terms.

(2)

The sum of the first n terms of the series is greater than 300.

(d) Calculate the smallest possible value of n.

(4)

A geometric series is $a + ar + ar^2 + \ldots$

(a) Prove that the sum of the first n terms of this series is given by

$$S_n = \frac{a(1-r^n)}{1-r}.$$

(4)

(b) Find

$$\sum_{k=1}^{10} 100(2^k).$$

(3)

(c) Find the sum to infinity of the geometric series

$$\frac{5}{6} + \frac{5}{18} + \frac{5}{54} + \ldots.$$

(3)

(d) State the condition for an infinite geometric series with common ratio r to be convergent.

(1)

Series P2

A trading company made a profit of £50 000 in 2006 (Year 1).

A model for future trading predicts that profits will increase year by year in a geometric sequence with common ratio r, $r > 1$.

The model therefore predicts that in 2007 (Year 2) a profit of £50 000r will be made.

(a) Write down an expression for the predicted profit in Year n.

(1)

The model predicts that in Year n, the profit made will exceed £200 000.

(b) Show that $n > \frac{\log 4}{\log r} + 1$.

(3)

Using the model with $r = 1.09$,

(c) find the year in which the profit made will first exceed £200 000,

(2)

(d) find the total of the profits that will be made by the company over the 10 years from 2006 to 2015 inclusive, giving your answer to the nearest £10 000.

(3)

The first three terms of a geometric series are $(k + 4)$, k and $(2k - 15)$ respectively, where k is a positive constant.

(a) Show that $k^2 - 7k - 60 = 0$.

(4)

(b) Hence show that $k = 12$.

(2)

(c) Find the common ratio of this series.

(2)

(d) Find the sum to infinity of this series.

(2)

Series P2

8) $a : 50{,}000$
$r = r$

$u_n = 50{,}000r^{n-1}$

b) $50{,}000r^{n-1} > 200{,}000$

$r^{n-1} > 4$

$\log r^{n-1} > \log 4$

$(n-1)\log r > \log 4$

$n-1 > \frac{\log 4}{\log r}$

$n > \frac{\log 4}{\log r} + 1$

c) $n > \frac{\log 4}{\log(1.09)} + 1$

$n > 16.086\ldots + 1$

$n > 17.086\ldots$ ← next positive whole integer.

$\therefore$ year 18

d) $2006 - 2015 = 10$ years

$S_{10} = \frac{a(1-r^{10})}{1-r} = \frac{50000(1-(1.09)^{10})}{1-1.09} = 759646.48\ldots$ to nearest £10,000

$\boxed{= £760{,}000}$

Q4a) $S_\infty = \frac{a}{1-r} = 480$

$\therefore \frac{120}{1-r} = 480 \Rightarrow \frac{1-r}{120} = \frac{1}{480}$

$\xRightarrow{\times 120} 1-r = \frac{120}{480} = \frac{1}{4}$

$\Rightarrow r = 1 - \frac{1}{4} = \boxed{\frac{3}{4}}$

b) $\left.\begin{array}{l} 5^{th} \text{ term} = ar^4 \\ 6^{th} \text{ term} = ar^5 \end{array}\right\}$ difference $= ar^4 - ar^5$

$= ar^4(1-r)$

$= 120 \times (\frac{3}{4})^4(1-\frac{3}{4})$

$= \boxed{9.49}$ (2dp)

c) $S_7 = \frac{a(1-r^n)}{1-r} = \frac{120(1-(\frac{3}{4})^7)}{1-\frac{3}{4}} = \boxed{416}$

d) $S_n > 300$

$\frac{120(1-(\frac{3}{4})^n)}{1-\frac{3}{4}} > 300$

$120(1-\frac{3}{4}^n) > \frac{300}{4}$

$1-(\frac{3}{4})^n > \frac{300}{4 \times 120}$

$1-(\frac{3}{4})^n > \frac{5}{8}$

$\therefore (\frac{3}{4})^n < 1 - \frac{5}{8}$

4d) $(\frac{3}{4})^n < \frac{3}{8}$

$\log(\frac{3}{4}^n) < \log(\frac{3}{8})$

$n \log(\frac{3}{4}) < \log(\frac{3}{8})$

$n > \frac{\log(\frac{3}{8})}{\log(\frac{3}{4})} \quad \therefore n > 3.409\ldots$

[signs change when ÷ by a negative number $\log\frac{3}{4} < 0$]

$\boxed{n_{min} = 4}$

a	ar	ar^2
$(k+4)$	k	$2k-15$

$\frac{ar}{a} = r = \frac{k}{k+4} \quad \therefore ar^2 = (k+4) \times \frac{(k^2)}{(k+4)^2}$

so $\frac{\cancel{(k+4)}(k^2)}{(k+4)^{\cancel{2}}} = 2k-15$

$\Rightarrow \frac{k^2}{k+4} = 2k-15$

$\Rightarrow k^2 = (2k-15)(k+4)$

$\Rightarrow k^2 = 2k^2 + 8k - 15k - 60$

$\Rightarrow k^2 - 7k - 60 = 0$

b) $(k-12)(k+5) = 0$

$k = 12$ or $k = -5$ ← reject as $k > 0$

$\therefore \boxed{k = 12}$

c) $r = \frac{k}{k+4} = \frac{12}{12+4} = \frac{12}{16} = \boxed{\frac{3}{4}}$

d) $S_\infty = \frac{a}{1-r} = \frac{k+4}{1-\frac{3}{4}} = \frac{12+4}{\frac{1}{4}} = \boxed{64}$

Q10a) $S_n = a + ar + ar^2 + \ldots + ar^{n-2} + ar^{n-1}$ — ①

$rS_n = ar + ar^2 + ar^3 + \ldots + ar^{n-1} + ar^n$ — ②

① − ②: $S_n - rS_n = a - ar^n$ [all terms cancel except a and ar^n]

$S_n(1-r) = a(1-r^n)$

$S_n = \frac{a(1-r^n)}{1-r}$

b) $\sum_{k=1}^{10} 100(2^k) \longrightarrow$ geometric series sum.

$a = 200$

$r = 2$

$200 + 200(2^1) + \ldots + 200(2^9)$

$\therefore \sum_{k=1}^{10} 100(2^k) = S_{10} = \frac{200(1-(2)^{10})}{1-2} = \boxed{204600}$

c) $\left.\begin{array}{ccc} \frac{5}{6} & \frac{5}{18} & \frac{5}{54} \\ a & ar & ar^2 \end{array}\right\} \begin{array}{l} a = \frac{5}{6} \\ r = \frac{1}{3} \end{array}$

$(\frac{5}{6} \times \frac{1}{3} = \frac{5}{18})$

$S_\infty = \frac{a}{1-r} = \frac{\frac{5}{6}}{1-\frac{1}{3}} = \boxed{\frac{5}{4}}$

d) $|r| < 1$ (ie $-1 < r < 1$)