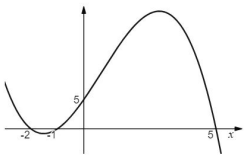
Year 13 A Level Maths Homework 1 Key Skills Due Date: _____ Name: _____ Score: /20

Answer all questions, make sure you show all working otherwise you will not be awarded marks. If you write on any other paper, please hand this in with the sheet.

1. Simplify $\frac{3x^2+16x+16}{9x+12}$	2. Simplify $\frac{x^2-x-12}{x^2-9} \times \frac{x^2-5x+6}{5}$	3. Simplify $\frac{x^2-2x}{x^2+2x} \div \frac{5}{x^2+5x+6}$	4. Simplify $2+\frac{x-5}{(x-3)(2x-1)}$	5. Simplify $\frac{5}{x+5}-\frac{3x}{(x+2)(x+5)}$
6. Simplify $\frac{10}{x^2+2x-24}-\frac{9}{x^2+3x-18}$	7. Given that (x - 2) is a factor, factorise $f(x)=3x^3-14x^2+32$	8. $f(x)=4x^3+8x^2+px-18$ Where p is a constant. Given (x + 2) is a factor, find p.	9. Factorise $f(x)=4x^3+4x^2-x-1$	10. Find the minimum point on the graph $y=x^2+8x+15$
11. Find the maximum point on the graph $y=-x^2-8x-14$	12. Sketch the graph of $y=x^2-6x+5$	13. Calculate the discriminant and state the number of real solutions $x^2-7x+4=0$	14. Given $g(x)=\frac{4x}{3}-5$, find $g^{-1}(x)$	15. The straight line $y=7px-8p$ touches the curve $y=2x^2+2x+2$ where p is a constant. Find the set of possible values of p.
16. Find the equation of the curve 	17. $(k-2)x-4x^2-4=0$ has equal roots, find the possible values of k.	18. The point P (0,2) lies on the curve with the equation y=f(x)). Find the image of P on the curve with equation y =f(x+2)	19. The point P (-7,4) lies on the curve with the equation y=f(x)). Find the image of P on the curve with equation y =-f(x)	20. Sketch the graph of $y=2x^3+7x^2-15x$

Mark scheme

Question 1

$\frac{x+4}{3}$

Factorise the numerator and denominator first, then you can divide top and bottom by the common factor.

$$\begin{aligned}\frac{3x^2+16x+16}{9x+12} &\equiv \frac{(x+4)(3x+4)}{3(3x+4)} \\ &\equiv \frac{(x+4)\cancel{(3x+4)}}{3\cancel{(3x+4)}} \\ &\equiv \frac{x+4}{3}\end{aligned}$$

Question 2

$\frac{(x-4)(x-2)}{5}$

① Factorise each term.

$$\frac{(x-4)(x+3)}{(x-3)(x+3)} \times \frac{(x-3)(x-2)}{5}$$

② Cancel out common factors.

$$\begin{aligned}&\frac{(x-4)\cancel{(x+3)}}{\cancel{(x-3)}\cancel{(x+3)}^{1}} \times \frac{\cancel{(x-3)}(x-2)}{5} \\ &= \frac{x-4}{1} \times \frac{x-2}{5}\end{aligned}$$

③ Multiply the fractions together.

$$\frac{(x-4)(x-2)}{5}$$

Question 3

$\frac{(x-2)(x+3)}{5}$

① Reciprocate (flip) the second fraction and multiply.

$$\frac{x^2-2x}{x^2+2x} \times \frac{x^2+5x+6}{5}$$

② Factorise each term.

$$\frac{x(x-2)}{x(x+2)} \times \frac{(x+2)(x+3)}{5}$$

③ Cancel out common factors.

$$\frac{x(x-2)}{x\cancel{(x+2)}^{1}} \times \frac{\cancel{(x+2)}(x+3)}{5}$$
$$= \frac{x-2}{1} \times \frac{x+3}{5}$$

④ Multiply the fractions together.

$$\frac{(x-2)(x+3)}{5}$$

Question 4

$$\frac{4x^2-13x+1}{(x-3)(2x-1)}$$

① Find the lowest common denominator.

$$2 + \frac{x-5}{(x-3)(2x-1)}$$
$$= \frac{2(x-3)(2x-1)}{(x-3)(2x-1)} + \frac{x-5}{(x-3)(2x-1)}$$
$$= \frac{4x^2-14x+6}{(x-3)(2x-1)} + \frac{x-5}{(x-3)(2x-1)}$$

② Add the numerators.

$$= \frac{4x^2-13x+1}{(x-3)(2x-1)}$$

Question 5

$$\frac{2}{x+2}$$

① Subtract the fractions.

$$\frac{5}{x+5} - \frac{3x}{(x+2)(x+5)}$$
$$= \frac{5(x+2)}{(x+2)(x+5)} - \frac{3x}{(x+2)(x+5)}$$
$$= \frac{5x+10}{(x+2)(x+5)} - \frac{3x}{(x+2)(x+5)}$$
$$= \frac{2x+10}{(x+2)(x+5)}$$

② Factorise the numerator and simplify.

$$= \frac{2(x+5)}{(x+2)(x+5)}$$
$$= \frac{2\cancel{(x+5)}}{(x+2)\cancel{(x+5)}}$$
$$= \frac{2}{x+2}$$

Question 6

$\frac{1}{(x-4)(x-3)}$

① Factorise the denominator.

$$\equiv \frac{10}{(x+6)(x-4)} - \frac{9}{(x+6)(x-3)}$$

② Multiply to get a common denominator.

$$\equiv \frac{10}{(x+6)(x-4)} \times \frac{x-3}{x-3} - \frac{9}{(x+6)(x-3)} \times \frac{x-4}{x-4}$$
$$\equiv \frac{10x-30}{(x+6)(x-4)(x-3)} - \frac{9x-36}{(x+6)(x-4)(x-3)}$$

③ Subtract the fractions and simplify the numerator.

$$\equiv \frac{x+6}{(x+6)(x-4)(x-3)}$$

④ Divide both the numerator and denominator by the common factor.

$$\equiv \frac{\cancel{x+6}}{\cancel{(x+6)}(x-4)(x-3)}$$
$$\equiv \frac{1}{(x-4)(x-3)}$$

Question 7

$(x-2)(x-4)(3x+4)$

① Divide $f(x)$ by $(x-2)$.

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② Factorise the quotient.

$$3x^2 - 8x - 16 \equiv (x-4)(3x+4)$$

③ Multiply by $(x-2)$.

$$f(x) = (x-2)(x-4)(3x+4)$$

Question 8

$p = -9$

$(x+2)$ is a factor of (Could not display math)

$$\begin{aligned} 4(-2)^3 + 8(-2)^2 + p(-2) - 18 &= 0 \\ -32 + 32 - 2p - 18 &= 0 \\ -2p - 18 &= 0 \\ p &= -9 \end{aligned}$$

Question 9

$(x+1)(2x+1)(2x-1)$

Using the factor theorem, $f(-1) = 0$ therefore $(x+1)$ is a factor.

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$4x^2 - 1 \equiv (2x+1)(2x-1)$

$\therefore 4x^3 + 4x^2 - x - 1$
$\equiv (x+1)(2x+1)(2x-1)$

Question 10

$(-4, -1)$

Complete the square:

$$\begin{aligned} y &= x^2 + 8x + 15 \\ &= (x+4)^2 - 16 + 15 \\ &= (x+4)^2 - 1 \end{aligned}$$

Therefore the minimum point is $(-4, -1)$

Question 11

$(-4, 2)$

Factorise the number in front of the x^2 term out of the first two terms

$y = -(x^2 + 8x) - 14$

Complete the square for the expression inside the bracket:

$= -((x+4)^2 - 16) - 14$

Expand out the outer bracket and simplify:

$$\begin{aligned} &= -(x+4)^2 + 16 - 14 \\ &= -(x+4)^2 + 2 \end{aligned}$$

Therefore the maximum point is $(-4, 2)$

Question 12

$a = 1, b = 5, c = 3, d = -4, e = 5$

① Find the x-intercepts

$$\begin{aligned} y &= x^2 - 6x + 5 \\ &= (x-1)(x-5) \\ x &= 1 \\ x &= 5 \end{aligned}$$

② Find the minimum point

$$\begin{aligned} y &= x^2 - 6x + 5 \\ &= (x-3)^2 - 9 + 5 \\ &= (x-3)^2 - 4 \end{aligned}$$

$\therefore$ minimum at $(3, -4)$

③ Find the y-intercept

$$\begin{aligned} y &= (0)^2 - 6(0) + 5 \\ &= 5 \end{aligned}$$

Question 13

Discriminant = 33
Number of real solutions = 2

① Identify the coefficients of a, b and c from the form $ax^2 + bx + c$.

$$\begin{aligned} a &= 1 \\ b &= -7 \\ c &= 4 \end{aligned}$$

② Substitute the values of a, b and c into the formula $b^2 - 4ac$.

$$(-7)^2 - 4 \times 1 \times 4 = 33$$

③ Since the discriminant is greater than 0, we have 2 solutions.

Question 14

$\frac{3(x+5)}{4}$

Replace $g(x)$ by y and make x the subject.

$$\begin{aligned} g(x) &= \frac{4x}{3} - 5 \\ y &= \frac{4x}{3} - 5 \\ y + 5 &= \frac{4x}{3} \\ 3(y+5) &= 4x \\ \frac{3(y+5)}{4} &= x \end{aligned}$$

Interchange x and y and then replace y by $g^{-1}(x)$

$$\begin{aligned} \frac{3(x+5)}{4} &= y \\ g^{-1}(x) &= \frac{3(x+5)}{4} \end{aligned}$$

Question 15

$p = -\frac{6}{49}$ or $p = 2$

① Equate the two expressions and make one side zero.

$$\begin{aligned} 2x^2 + 2x + 2 &= 7px - 8p \\ (2)x^2 + (-7p+2)x + (8p+2) &= 0 \end{aligned}$$

② "touches the curve" implies that there is one point of intersection, and that the discriminant is equal to zero.

Solve $\Delta = 0$ where $\Delta = b^2 - 4ac$

$$\begin{aligned} (-7p+2)^2 - 4(2)(8p+2) &= 0 \\ 49p^2 - 92p - 12 &= 0 \\ p = -\frac{6}{49} \text{ or } p &= 2 \end{aligned}$$

Question 16

$a = -\frac{1}{2}, b = 1, c = \frac{13}{2}, d = 5$

① Use the roots to find an initial equation.

$$y = a(x+1)(x+2)(5-x)$$

② Substitute $x = 0$ and equate with the y-intercept.

$$\begin{aligned} a \times 1 \times 2 \times 5 &= 5 \\ a &= \frac{1}{2} \end{aligned}$$

③ Expand and simplify.

$y = \frac{1}{2}(x+1)(x+2)(5-x)$

$y = -\frac{1}{2}x^3 + x^2 + \frac{13}{2}x + 5$

Question 17

$k = 10$ or $k = -6$

① Identify the coefficients of a, b and c from the form $ax^2 + bx + c$

$a = -4$
$b = k - 2$
$c = -4$

② Substitute the values of a, b and c into $b^2 - 4ac$

$(k-2)^2 - 4 \times (-4) \times (-4)$

③ Expand and simplify.

$(k^2 - 4k + 4) - 64$
$= k^2 - 4k - 60$

④ Equate to 0 and solve.

$k^2 - 4k - 60 = 0$
$k = 10$
$k = -6$

Question 18

$(-2, 2)$

$y = f(x+2)$ is a translation by vector $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$

Therefore the image of P is $(-2, 2)$

Question 19

$(-7, -4)$

$y = -f(x)$ is a reflection in the x-axis which means that the y-coordinate is multiplied by -1.

Therefore the image of P is $(-7, -4)$

Question 20

$a = -5, b = 0, c = \frac{3}{2}$

① Factorise.

$$\begin{aligned} & 2x^3 + 7x^2 - 15x \\ = \ & x(2x^2 + 7x - 15) \\ = \ & x(2x - 3)(x + 5) \end{aligned}$$

② Solve $x(2x - 3)(x + 5) = 0$.

$$\begin{aligned} x &= \frac{3}{2} \\ x &= 0 \\ x &= -5 \end{aligned}$$

③ Find the y-intercept by substituting $x = 0$.

$$-3 \times 0 \times 5 = 0$$
