Differentiation P2

Find the coordinates of the stationary point on the curve with equation $y = 2x^2 - 12x$.

(4)

The curve C has equation

$$y = 2x^3 - 5x^2 - 4x + 2.$$

(a) Find $\dfrac{dy}{dx}$.

(2)

(b) Using the result from part (a), find the coordinates of the turning points of C.

(4)

(c) Find $\dfrac{d^2y}{dx^2}$.

(2)

(d) Hence, or otherwise, determine the nature of the turning points of C.

(2)

A diesel lorry is driven from Birmingham to Bury at a steady speed of v kilometres per hour. The total cost of the journey, £C, is given by

$$C = \frac{1400}{v} + \frac{2v}{7}.$$

(a) Find the value of v for which C is a minimum.

(5)

(b) Find $\dfrac{d^2C}{dv^2}$ and hence verify that C is a minimum for this value of v.

(2)

(c) Calculate the minimum total cost of the journey.

(2)

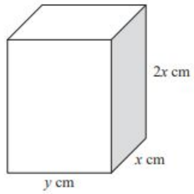

Figure 4

Figure 4 shows a solid brick in the shape of a cuboid measuring $2x$ cm by x cm by y cm.

The total surface area of the brick is 600 cm^2.

(a) Show that the volume, V cm^3, of the brick is given by

$$V = 200x - \frac{4x^3}{3}.$$

(4)

Given that x can vary,

(b) use calculus to find the maximum value of V, giving your answer to the nearest cm^3.

(5)

(c) Justify that the value of V you have found is a maximum.

(2)

Differentiation P2

1) $y = 2x^2 - 12x$

$\frac{dy}{dx} = 4x - 12$

when $\frac{dy}{dx} = 0$ $\quad 4x = 12$

$\underline{x = 3}$

when $x = 3$

$y = 2(3)^2 - 12(3)$

$y = -18$

$\boxed{(3, -18)}$

Q7a) $\frac{dy}{dx} = 6x^2 - 10x - 4$

b) $6x^2 - 10x - 4 = 0$

$\Rightarrow 3x^2 - 5x - 2 = 0$

$\Rightarrow (3x + 1)(x - 2) = 0$

$\therefore 3x + 1 = 0$ $\quad x = -\frac{1}{3}$ | $x - 2 = 0$ $\quad x = 2$

$y = 2(-\frac{1}{3})^3 - 5(-\frac{1}{3})^2 - 4(-\frac{1}{3}) + 2$

$= \frac{73}{27}$ //

$y = 2(2)^3 - 5(2)^2 - 4(2) + 2$

$= -10$ //

so $(-\frac{1}{3}, \frac{73}{27})$ and $(2, -10)$ are the turning points.

Q8a) $C = 1400v^{-1} + \frac{2}{7}v$

$\frac{dC}{dv} = -1400v^{-2} + \frac{2}{7} = 0$

$\frac{2}{7} = \frac{1400}{v^2}$

$v^2 = 4900 \quad \therefore v = \sqrt{4900} = \boxed{70 \text{ km/h}}$

b) $\frac{d^2C}{dv^2} = 2800v^{-3} > 0$ (for all values of $v > 0$)

c) $C = \frac{1400}{(70)} + \frac{2}{7}(70) = \boxed{£40}$

10a) surface area $= 2 \times 2x^2 + 2 \times 2xy + 2xy$

$= 4x^2 + 6xy$

$600 = 4x^2 + 6xy$

$\frac{600 - 4x^2}{6x} = y$

$V = 2x^2 y = 2x^2\left(\frac{600 - 4x^2}{6x}\right)$

$V = \frac{1200x^2 - 8x^4}{6x} = 200x - \frac{4}{3}x^3$

b) $\frac{dV}{dx} = 200 - 4x^2$

$\frac{dV}{dx} = 0$ $\quad 200 = 4x^2$

$x^2 = 50$

$x = \pm\sqrt{50}$

$x = \sqrt{50}$

$V = 200(\sqrt{50}) - \frac{4}{3}(\sqrt{50})^3 \doteq 943 \text{ cm}^3$

c) $\frac{d^2V}{dx^2} = -8x$

$-8x < 0 \quad -8\sqrt{50} < 0$

$\therefore \frac{d^2V}{dx^2} < 0 \quad \therefore$ it is a maximum