Mechanics 1

$$\text{Average velocity} = \frac{\text{displacement from starting point}}{\text{time taken}}$$

$$\text{Average speed} = \frac{\text{total distance travelled}}{\text{time taken}}$$

$$\text{Velocity (final)} = \text{Velocity (initial)} + (\text{Acceleration} \times \text{Time}) \quad [v = u + at]$$

$$\text{Displacement} = \frac{\text{Velocity (initial)} + \text{Velocity (final)}}{2} \times \text{Time} \quad \left[s = \left(\frac{u+v}{2}\right)t\right]$$

$$\text{Velocity (final)}^2 = \text{Velocity (initial)}^2 + (2 \times \text{Acceleration} \times \text{Displacement}) \quad [v^2 = u^2 + 2as]$$

$$\text{Displacement} = \text{Velocity (initial)} \times \text{Time} + \frac{1}{2} \times \text{Acceleration} \times \text{Time}^2 \quad \left[s = ut + \frac{1}{2}at^2\right]$$

$$\text{Displacement} = \text{Velocity (final)} \times \text{Time} - \frac{1}{2} \times \text{Acceleration} \times \text{Time}^2 \quad \left[s = vt - \frac{1}{2}at^2\right]$$

$$\text{Force} = \text{Mass} \times \text{Acceleration} \quad [F = ma] \qquad [F_{MAX} = \mu R]$$

$$\sin\theta = \frac{\text{Opposite}}{\text{Hypotenuse}}, \quad \cos\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}, \quad \tan\theta = \frac{\text{Opposite}}{\text{Adjacent}}$$

Maximum/limiting value of friction = Coefficient of friction × Normal reaction force

$$\text{Momentum before collision} = \text{Momentum after collision} \quad [m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2]$$

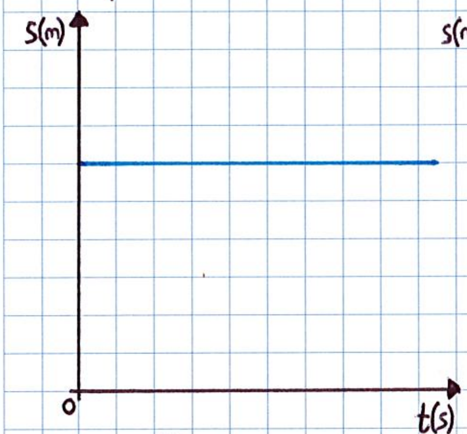
$$\text{Momentum} = \text{Mass} \times \text{Velocity} \quad [P = mv]$$

$$\text{Impulse} = \text{Force} \times \text{Time} \quad [I = Ft]$$

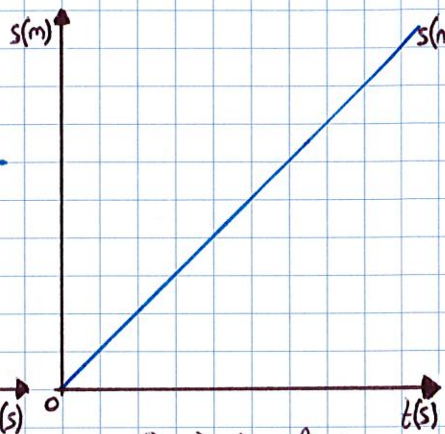
$$= \text{Final momentum} - \text{Initial momentum} \quad [I = mv - mu]$$

Graphs

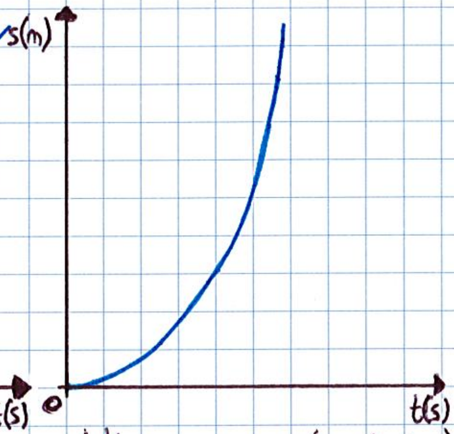
Displacement-time



Stationary

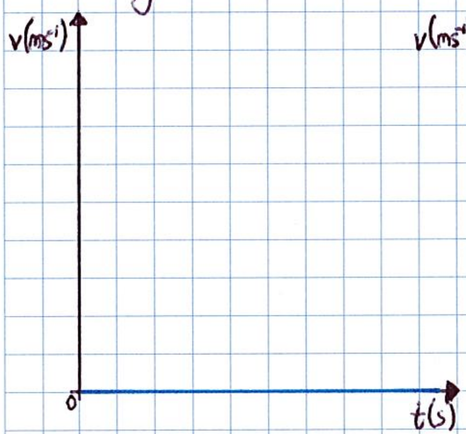


Constant velocity

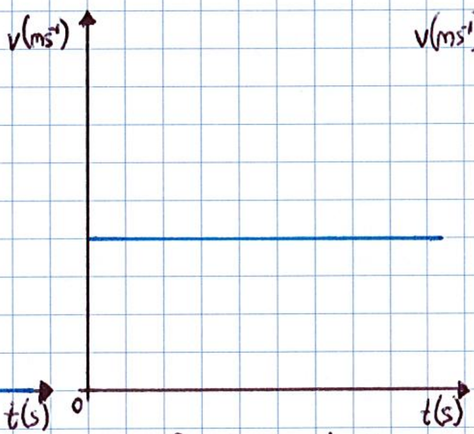


Velocity increases (accelerating)

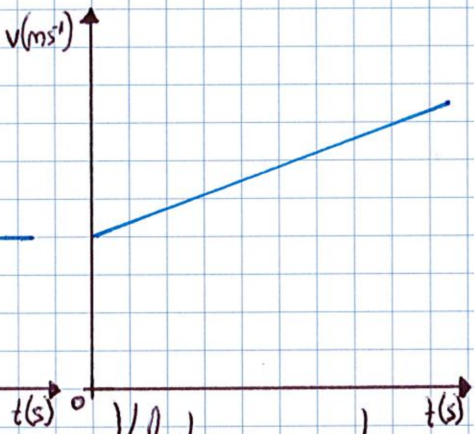
Velocity-time



Stationary

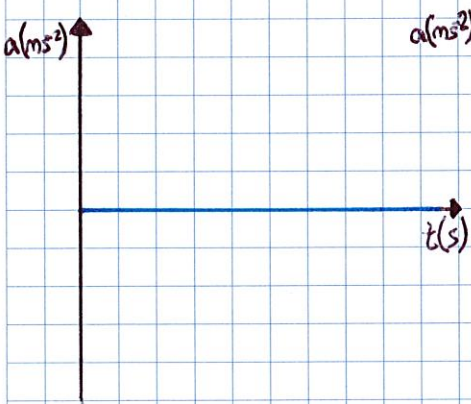


Constant velocity

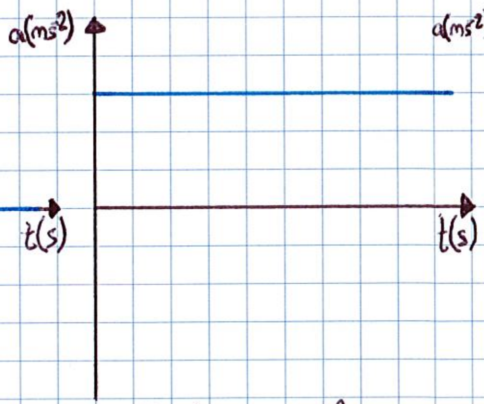


Velocity increases at a constant rate (constant acceleration)

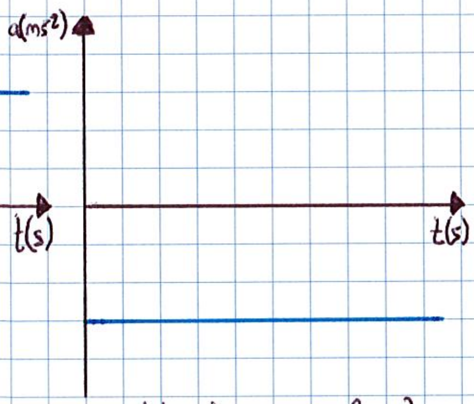
Acceleration-time



Zero acceleration
(Constant velocity)



Positive acceleration
(Increase in velocity)



Negative acceleration
(Decrease in velocity)

Model	Modelling assumptions
Particle - Dimensions of the obbject are **negligible**.	· **mass** of the object is concentrated at a single point · rotational forces (i.e. moving around a central fixed point) and air **resistance** can be ignored
Rod - All dimensions but one are negligible, like a pole or a beam.	· mass is concentrated along a line · no thickness · **rigid** (does not bend or buckle)
Lamina - Object with area but negligible thickness, like a sheet of paper.	· mass is distributed across a flat surface
Uniform body - Mass is distributed evenly.	· mass of the object is concentrated at single point at the geometric centre of theb **body** - the **center of mass**
Light object - Mass of the object is small compared to other masses, like a string or a pulley.	· treat object as having zero mass · **tension** the same at both ends of a light string
Inextensible string - A string that does not stretch under load.	· **acceleration** is the same in objects connected by a **taut** inextensible string
Smooth surface - A surface on which it can be assumed there is no friction.	· assume that there is no **friction** between the surface and any object on it
Rough surface - a surface on which there is friction.	· objects in contact with the surface experience a frictional force if the are moving, or are acted on by a force
Wire - Rigid thin length of metal.	· treated as one-dimensional
Smooth and light pulley - All pulleys you consider will be smooth and light.	· pulley has no mass · tension is the same on either side of the pulley
Bead - Particle with a hole in it for threading on a wire or string (i.e. passing the wire or string through the hole).	· a smooth bead moves freely along a wire or string · for a smooth bead, tension is the same on either side of the bead
Peg - A support from which a body can be suspended or rested.	· dimensionless and fixed · can be rough or smooth as specified in the question
Air resistance - Resistance experienced as an object moves through the air.	· usually modelled as being negligible
Gravity - Force of attraction between all objects. Acceleration due to gravity is denoted by *g*.	· assume all objects with mass are attracted toward the Earth · acceleration due to Earth's gravity is uniform (i.e. the same in all parts, at all times) and acts vertically downward · *g* is **constant** and is taken as 9.8 m s^(-2), unless otherwise stated in the question

Quantity	Unit	Symbol
Mass	kilogram	kg
Length/**displacement**	meter	m
Time	second	s

Quantity	Unit	Symbol
Speed/velocity	meters per second	m s^(-1)
Acceleration	meters per second per second	m s^(-2)
Weight/force	newton	N (= kg m s^(-2))

Mechanics 2

The horizontal motion of a projectile is modelled as having constant velocity ($a = 0$). You can use the formula $s = vt$.

The vertical motion of a projectile is modelled as having constant acceleration due to gravity ($a = g$).

When a particle is projected with initial velocity U at an angle α above the horizontal, and that moves freely under gravity:

- The horizontal component of the initial velocity is $U\cos\alpha$.
- The vertical component of the initial velocity is $U\sin\alpha$.
- Time of flight: $\dfrac{2U\sin\alpha}{g}$
- Time to reach greatest height: $\dfrac{U\sin\alpha}{g}$
- Range on horizontal plane: $\dfrac{U^2\sin 2\alpha}{g}$
- Equation of trajectory: $y = x\tan\alpha - gx^2\dfrac{1+\tan^2\alpha}{2U^2}$

where y is the vertical height of the particle, x is the horizontal distance from the point of projection, and g is the acceleration due to gravity

$$\text{displacement} = s = \int v\,dt$$

Differentiate

$$\frac{ds}{dt} = \text{velocity} = v = \int a\,dt$$

Integrate

$$\frac{dv}{dt} = \frac{d^2s}{dt^2} = \text{acceleration} = a$$

Work done by a Force = Component of Force in direction of motion × Distance moved in direction of Force $[W = Fs]$

Work done against gravity = Mass × Acceleration due to gravity × Vertical distance raised
≡ Potential energy (P.E) $[E_p = W = mgh]$

Kinetic energy (K.E) = $\frac{1}{2}$ × Mass of the particle × Speed $[E_k = \frac{1}{2}mv^2]$

For a particle moving horizontally, work done = change in Kinetic energy.

<u>Principle of conservation of mechanical energy:</u> When no external forces (other than gravity) do work on a particle during its motion, the sum of the particle's kinetic and potential energy remains constant.

<u>Work-energy principle:</u> The change in the total energy of a particle is equal to the work done on the particle.

Power is the rate of doing work. Power = Force × Speed $[P = Fv]$

Newton's law of restitution: $\frac{\text{speed of separation of particles}}{\text{speed of approach of particles}} = e$

where the constant e is the coefficient of restitution between the particles; $0 \leqslant e \leqslant 1$.

The loss of Kinetic energy due to impact $= \left(\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2\right) - \left(\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2\right)$

Moment of a Force acting at a perpendicular distance from a point is given by Force × Distance. [Moment of a Force = Fd]

Clockwise moment of F about $P = |F| \times d\sin\theta$