

Figure 1

The circle C with centre T and radius r has equation

$$x^2 + y^2 - 20x - 16y + 139 = 0$$

- (a) Find the coordinates of the centre of C .

(3)

- (b) Show that $r = 5$

(2)

The line L has equation $x = 13$ and crosses C at the points P and Q as shown in Figure 1.

- (c) Find the y coordinate of P and the y coordinate of Q .

(3)

The circle C has equation

$$x^2 + y^2 - 20x - 24y + 195 = 0$$

The centre of C is at the point M .

- (a) Find

- (i) the coordinates of the point M ,

- (ii) the radius of the circle C .

(5)

N is the point with coordinates $(25, 32)$.

- (b) Find the length of the line MN .

(2)

The tangent to C at a point P on the circle passes through point N .

- (c) Find the length of the line NP .

(2)

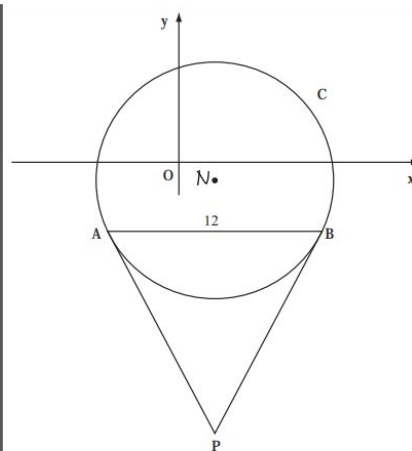


Figure 3

Figure 3 shows a sketch of the circle C with centre N and equation

$$(x - 2)^2 + (y + 1)^2 = \frac{169}{4}$$

- (a) Write down the coordinates of N .

(2)

- (b) Find the radius of C .

(1)

The chord AB of C is parallel to the x -axis, lies below the x -axis and is of length 12 units as shown in Figure 3.

- (c) Find the coordinates of A and the coordinates of B

(5)

- (d) Show that angle $ANB = 134.8^\circ$, to the nearest 0.1 of a degree.

(2)

The tangents to C at the points A and B meet at the point P

- (e) Find the length AP , giving your answer to 3 significant figures.

(2)

Circles P2

The circle C has centre $A(2, 1)$ and passes through the point $B(10, 7)$.

- (a) Find an equation for C .

(4)

The line l_1 is the tangent to C at the point B .

- (b) Find an equation for l_1 .

(4)

The line l_2 is parallel to l_1 and passes through the mid-point of AB .

Given that l_2 intersects C at the points P and Q ,

- (c) find the length of PQ , giving your answer in its simplest surd form.

(3)

$$3a) x^2 + y^2 - 20x - 16y + 139 = 0$$

$$(x-10)^2 - 100 + (y-8)^2 - 64 + 139 = 0$$

$$(x-10)^2 + (y-8)^2 - 25 = 0$$

$$(x-10)^2 + (y-8)^2 = 5^2$$

$$\text{centre} = (10, 8)$$

$$c) (x-10)^2 + (y-8)^2 = 25$$

$$\text{when } x=13$$

$$(3)^2 + (y-8)^2 = 25$$

$$y^2 - 16y + 64 = 16$$

$$y^2 - 16y + 48 = 0$$

$$(y-4)(y-12)$$

$$y=4 \text{ or } y=12$$

$$d) \text{ arc length} = r\theta = 5 \times 1.855 = 9.275$$

$$9.275 + 2r = 9.275 + 10 = 19.275$$

5i) completing square

$$(x-10)^2 - 100 + (y-12)^2 - 144 + 195 = 0$$

$$(x-10)^2 + (y-12)^2 = 49$$

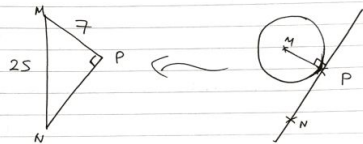
$$\therefore \text{centre} : (10, 12)$$

$$\text{radius} = \sqrt{49} = 7$$

ii)

$$b) |MN| = \sqrt{(25-10)^2 + (32-12)^2} = 25 \text{ units}$$

c)



$$|PN| = \sqrt{25^2 - 7^2} = 24 \text{ units}$$

Circles P2

$$Q12a) \text{ radius} = |AB| = \sqrt{(6-2)^2 + (7-1)^2} = 10$$

$$\therefore (x-2)^2 + (y-1)^2 = 100$$

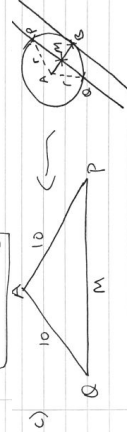
$$b) M_{AB} = \frac{7-1}{10-2} = \frac{6}{8} = \frac{3}{4}$$

\therefore gradient of tangent will be $-\frac{4}{3}$
[tangent is perpendicular to AB]

$$y-7 = -\frac{4}{3}(x-10)$$

$$y = -\frac{4}{3}x + \frac{40}{3} + 7$$

$$y = -\frac{4}{3}x + \frac{61}{3}$$



let M be midpoint of AB,

$$M = \left(\frac{2+10}{2}, \frac{1+7}{2} \right) = (6, 4)$$

$$\therefore |AM| = \sqrt{(6-2)^2 + (4-1)^2} = 5$$

now consider $\triangle AMQ$:

$$|MQ| = \sqrt{(10)^2 - (5)^2} = 5\sqrt{3}, \text{ and } |MQ| = |MP|$$

so distance from P to Q = $2 \times 5\sqrt{3}$

$$= 10\sqrt{3}$$

Q8b) N(2, -1)

$$b) r = \sqrt{\frac{16}{4}} = \frac{13}{2}$$

c) $\sqrt{\left(\frac{13}{2}\right)^2 - 10^2} = \frac{5}{2}$
A is 6 units to the left and 2.5 units down from N.

$$\therefore A\left(-4, -\frac{5}{2}\right) \text{ and } B\left(8, -\frac{5}{2}\right)$$

$$d) \text{ cosine rule: } \cos \angle ANB = \frac{\left(\frac{13}{2}\right)^2 + \left(\frac{13}{2}\right)^2 - (12)^2}{2 \left(\frac{13}{2}\right) \left(\frac{13}{2}\right)} = \frac{-119}{169}$$

$$\angle ANB = \cos^{-1}\left(\frac{-119}{169}\right) = 134.8^\circ$$

e)

