Given that $\log_3 x = a$, find in terms of $a$,

(a) $\log_3 (9x)$

**(2)**

(b) $\log_3 \left(\frac{x^5}{81}\right)$

**(3)**

giving each answer in its simplest form.

(c) Solve, for $x$,

$$\log_3(9x) + \log_3\left(\frac{x^5}{81}\right) = 3$$

giving your answer to 4 significant figures.

**(4)**

Given that $0 < x < 4$ and

$$\log_5(4-x) - 2\log_5 x = 1,$$

find the value of $x$.

**(6)**

Logs and Exponentials

Given that $a$ and $b$ are positive constants, solve the simultaneous equations

$$a = 3b,$$

$$\log_3 a + \log_3 b = 2.$$

Give your answers as exact numbers.

**(6)**

(a) Given that

$$2\log_3(\mathbf{x}-5) - \log_3(2\mathbf{x}-13) = 1,$$

show that $\mathbf{x}^2 - 16\mathbf{x} + 64 = 0$.

**(5)**

(b) Hence, or otherwise, solve $2\log_3(\mathbf{x}-5) - \log_3(2\mathbf{x}-13) = 1$.

**(2)**

6a) $\log_3 x = a$

$\cancel{\log_3 9x}$

$\therefore \log_3(9x) = \log_3(9) + \log_3(x)$

$= \boxed{2 + a}$

b) $\log_3\left(\frac{x^5}{81}\right) = \log_3(x^5) - \log_3(81)$

$= 5\log_3(x) - 4$

$= \boxed{5a - 4}$

c) $\log_3(9x) + \log_3\left(\frac{x^5}{81}\right) = 3$

from (a) and (b),

$(2+a) + (5a-4) = 3$

$6a - 2 = 3$

$6a = 5 \quad \therefore \boxed{a = \frac{5}{6}}$

and $\log_3 x = a$

$\therefore 3^a = x$

$\Rightarrow x = 3^{\frac{5}{6}} = \boxed{2.498}$

Q4) $\log_5(4-x) - \log_5(x^2) = 1$

$\log_5\left(\frac{4-x}{x^2}\right) = 1$

$5^1 = \frac{4-x}{x^2}$

$5x^2 = 4 - x$

$5x^2 + x - 4 = 0$ //

$(5x - 4)(x+1) = 0 \quad \therefore \boxed{x = \frac{4}{5}}$ (reject $x = -1$ as $0 < x < 4$)

Logs and Exponentials

$a = 3b$ — ①

$\log_3 a + \log_3 b = 2$ — ②

using ② : $\log_3(ab) = 2$

$3^2 = ab = 9$

from ① : $a = 3b$ // $\therefore 3b^2 = 9$

$b^2 = 3 \quad \therefore \boxed{b = \sqrt{3}}$ //

$a = 3b = \boxed{3\sqrt{3}}$

a) $\log_3[(x-5)^2] - \log_3[2x-13] = 1$

$\log_3\left[\frac{(x-5)^2}{2x-13}\right] = 1$

$3^1 = \frac{(x-5)^2}{2x-13}$

$6x - 39 = x^2 - 10x + 25$

$x^2 - 16x + 25 + 39 = 0$

$x^2 - 16x + 64 = 0$

b) $(x-8)^2 = 0$

$\boxed{x = 8}$