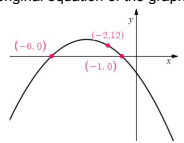
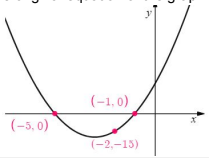
Year 12 AS Level Maths Homework Key Skills 5: Due Date: _____ Name: _____ Score: /30

Answer all questions, <u>make sure you show all working otherwise you will not be awarded marks</u>. If you write on any other paper, please hand this in with the sheet.

1. Solve $2x+9 \geq -5$	2. Solve $10 > 8a+2$	3. Solve $5y+8 \leq 9y+4$	4. Solve $3 \leq 4x+13 \leq 22$	5. Solve $8-3x \leq -3$	6.Solve $6(x+2)-11 < 4(x+3)$
7. Solve $x^2+4x \leq 0$	8. Solve $x^2+3x-10 \leq 0$	9. Solve $x(x+1) \geq 6$	10. Solve $x^2 \leq 5x$	11. Solve $z(z+6)+5 < 0$	12. Solve $9x^2 \leq 49$
13. Solve $(5z+4)(z-3)+14 > 0$	14. Solve $7x-12-x^2 > 0$	15. Solve $4-z^2 \geq 0$	16. Solve $1 \geq \frac{7x}{2x^2+5}$	17. n is an integer that satisfies both: $5n+2 < 12$ and $2n^2-5n+2 \leq 0$ Find the possible values of n	18. n is an integer that satisfies both: $4n+7 > 27$ and $n^2-7n+6 \leq 0$ Find the possible values of n
19. Solve $y = x^2+12x+24$, $y = 2x+3$	20. Solve $y = x^2+7x+17$, $y = -2x+3$	21. Solve $y = 4x+5$, $x^2+y^2 = 58$	22. Solve $x+y = 5$, $x^2+y^2 = 25$	23. Solve $x-y = 5$, $x^2+y^2 = 25$	24. Find the original equation of the graph 
25. Find the original equation of the graph 	26. Find the range of f(x) $f(x) = 3x^2-7x-4, \quad x \in \mathbb{R}$	27. Find fg(10) $f(x) = \frac{10x+10}{4x+10}$, $g(x) = \frac{5x-6}{4x-7}$	28. Find gg(x) $g(x) = 10x+10$	29. Given $f(x) = 4x+5$, find $f^{-1}(x)$.	30. Given $g(x) = \sqrt{\frac{6x-7}{10x+1}}$, find $g^{-1}(x)$.

Mark scheme

Question 1

$x \geq -7$

$$\begin{aligned} 2x + 9 &\geq -5 \\ -9 \downarrow &\quad \downarrow -9 \\ 2x &\geq -14 \\ \div 2 \downarrow &\quad \downarrow \div 2 \\ x &\geq -7 \end{aligned}$$

Question 2

$a < 1$

$$\begin{aligned} 10 &> 8a + 2 \\ -2 \downarrow &\quad \downarrow -2 \\ 8 &> 8a \\ \div 8 \downarrow &\quad \downarrow \div 8 \\ 1 &> a \\ \therefore a &< 1 \end{aligned}$$

Question 3

$y \geq 1$

$$\begin{aligned} 5y + 8 &\leq 9y + 4 \\ -5y \downarrow &\quad \downarrow -5y \\ 8 &\leq 4y + 4 \\ -4 \downarrow &\quad \downarrow -4 \\ 4 &\leq 4y \\ \div 4 \downarrow &\quad \downarrow \div 4 \\ 1 &\leq y \\ \therefore y &\geq 1 \end{aligned}$$

Question 4

$-\frac{5}{2} \leq x \leq \frac{9}{4}$

$$\begin{aligned} 3 \leq\ & 4x + 13 \leq\ 22 \\ -13\ & -13 - 13 \\ -10 \leq\ & 4x \leq\ 9 \\ \div\ 4\ & \div\ 4\ \div\ 4 \\ -\tfrac{5}{2} \leq\ & x \leq\ \tfrac{9}{4} \end{aligned}$$

Question 5

$x \geq \frac{11}{3}$

$$\begin{aligned} 8 - 3x &\leq -3 \\ +3x \downarrow &\quad \downarrow +3x \\ 8 &\leq 3x - 3 \\ +3 \downarrow &\quad \downarrow +3 \\ 11 &\leq 3x \\ \div 3 \downarrow &\quad \downarrow \div 3 \\ \frac{11}{3} &\leq x \\ \therefore x &\geq \frac{11}{3} \end{aligned}$$

Question 6

$x < \frac{11}{2}$

① Expand the brackets and simplify.

$$\begin{aligned} 6(x+2) - 11 &< 4(x+3) \\ 6x + 12 - 11 &< 4x + 12 \\ 6x + 1 &< 4x + 12 \end{aligned}$$

② Solve the inequality.

$$\begin{aligned} 6x + 1 &< 4x + 12 \\ -4x \downarrow &\quad \downarrow -4x \\ 2x + 1 &< 12 \\ -1 \downarrow &\quad \downarrow -1 \\ 2x &< 11 \\ \div 2 \downarrow &\quad \downarrow \div 2 \\ x &< \frac{11}{2} \end{aligned}$$

Question 7

$-4 \leq x \leq 0$

âž€ Factorise and find the critical values.

$$\begin{aligned} x^2 + 4x &\leq 0 \\ x(x+4) &\leq 0 \end{aligned}$$

$x = -4$ or $x = 0$

âž To solve $x(x+4) \leq 0$, sketch $y = x(x+4)$, then consider where $y \leq 0$

Therefore $-4 \leq x \leq 0$

Question 8

$-5 \leq x \leq 2$

â€¢ Factorise and find the critical values.

$$x^2 + 3x - 10 \leq 0$$
$$(x+5)(x-2) \leq 0$$
$$x = -5 \text{ or } x = 2$$

â To solve $(x+5)(x-2) \leq 0$, sketch $y = (x+5)(x-2)$, then consider where $y \leq 0$

Therefore $-5 \leq x \leq 2$

Question 9

$x \leq -3$ or $x \geq 2$

â€¢ Expand the brackets and make one side of the inequality 0.

$$x(x+1) \geq 6$$
$$x^2 + x \geq 6$$
$$x^2 + x - 6 \geq 0$$

â Factorise and find the critical values.

$$(x+3)(x-2) \geq 0$$
$$x = -3 \text{ or } x = 2$$

â, To solve $(x+3)(x-2) \geq 0$, sketch $y = (x+3)(x-2)$, then consider where $y \geq 0$

Therefore $x \leq -3$ or $x \geq 2$

Question 10

$0 \leq x \leq 5$

â€¢ Make one side of the inequality 0.

$$x^2 \leq 5x$$
$$x^2 - 5x \leq 0$$

â Factorise and find the critical values.

$$x(x-5) \leq 0$$
$$x = 0 \text{ or } x = 5$$

â, To solve $x(x-5) \leq 0$, sketch $y = x(x-5)$, then consider where $y \leq 0$

Therefore $0 \leq x \leq 5$

Question 11

$-5 < z < -1$

â€¢ Expand the brackets.

$$z(z+6)+5<0$$
$$z^2+6z+5<0$$

â Factorise and find the critical values.

$$(z+5)(z+1)<0$$
$z = -5$ or $z = -1$

â, To solve $(z+5)(z+1)<0$, sketch $y = (z+5)(z+1)$, then consider where $y < 0$

Therefore $-5 < z < -1$

Question 12

$-\frac{7}{3} \leq x \leq \frac{7}{3}$

â€¢ Make one side of the inequality 0.

$$9x^2 \leq 49$$
$$9x^2-49 \leq 0$$

â Factorise by the difference of two squares and find the critical values.

$$(3x+7)(3x-7) \leq 0$$
$x = -\frac{7}{3}$ or $x = \frac{7}{3}$

â To solve $(3x+7)(3x-7) \leq 0$, sketch $y = (3x+7)(3x-7)$, then consider where $y \leq 0$

Therefore $-\frac{7}{3} \leq x \leq \frac{7}{3}$

Question 13

$z < \frac{1}{5}$ or $z > 2$

â€¢ Expand the brackets and simplify.

$$(5z+4)(z-3)+14>0$$
$$5z^2-11z-12+14>0$$
$$5z^2-11z+2>0$$

âž Factorise and find the critical values.

$$(5z-1)(z-2)>0$$
$$z=\frac{1}{5} \text{ or } z=2$$

âž, To solve $(5z-1)(z-2)>0$, sketch $y=(5z-1)(z-2)$, then consider where $y>0$

Therefore $z<\frac{1}{5}$ or $z>2$

Question 14

$$3<x<4$$

â€ Multiply the equation by -1 to get a positive x^2 term which also flips the inequality sign.

$$7x-12-x^2>0$$
$$12-7x+x^2<0$$
$$x^2-7x+12<0$$

âž Factorise and find the critical values.

$$(x-3)(x-4)<0$$
$$x=3 \text{ or } x=4$$

âž, To solve $(x-3)(x-4)<0$, sketch $y=(x-3)(x-4)$, then consider where $y<0$

Therefore $3<x<4$

Question 15

$$-2\leq z\leq 2$$

â€ Multiply the equation by -1 to get a positive x^2 term which also flips the inequality sign.

$$4-z^2\geq 0$$
$$-4+z^2\leq 0$$
$$z^2-4\leq 0$$

âž Factorise and find the critical values.

$$(z+2)(z-2)\leq 0$$
$$z=-2 \text{ or } z=2$$

â, To solve $(z+2)(z-2) \leq 0$, sketch $y = (z+2)(z-2)$, then consider where $y \leq 0$

Therefore $-2 \leq z \leq 2$

Question 16

$x \leq 1$ or $x \geq \frac{5}{2}$

â€ Given that $2x^2+5$ is always positive, multiply by $2x^2+5$ and rearrange.

$$\begin{aligned} 1 &\geq \frac{7x}{2x^2+5} \\ 2x^2+5 &\geq 7x \\ 2x^2-7x+5 &\geq 0 \end{aligned}$$

â Factorise and find the critical values.

$$\begin{aligned} 2x^2-7x+5 &\geq 0 \\ (x-1)(2x-5) &\geq 0 \\ x = 1 \text{ or } x &= \frac{5}{2} \end{aligned}$$

â, Draw a graph and solve.

Therefore $x \leq 1$ or $x \geq \frac{5}{2}$

Question 17

1

â€ Solve the first inequality.

$$\begin{aligned} 5n+2 &< 12 \\ 5n &< 10 \\ n &< 2 \end{aligned}$$

â Factorise and find the critical values.

$$\begin{aligned} 2n^2-5n+2 &\leq 0 \\ (2n-1)(n-2) &\leq 0 \\ n = \frac{1}{2} \text{ or } n &= 2 \end{aligned}$$

â, Draw a graph and solve.

Therefore $\frac{1}{2} \leq n \leq 2$

âƒ Combine the two sets of solutions.

$\frac{1}{2} \leq n \leq 2$ and $n < 2$

$\frac{1}{2} \leq n < 2$
$n = 1$

Question 18

6

âž€ Solve the first inequality.

$4n + 7 > 27$
$4n > 20$
$n > 5$

âž Factorise and find the critical values.

$n^2 - 7n + 6 \leq 0$
$(n-1)(n-6) \leq 0$
$n = 1$ or $n = 6$

âž, Draw a graph and solve.

Therefore $1 \leq n \leq 6$

âžƒ Combine the two sets of solutions.

$1 \leq n \leq 6$ and $n > 5$

$5 < n \leq 6$
$n = 6$

Question 19

$x = -7, y = -11$ or $x = -3, y = -3$

Equate the right-handside of the two equations and solve the resulting quadratic equation.

$x^2 + 12x + 24 = 2x + 3$
$x^2 + 10x + 21 = 0$
$(x+7)(x+3) = 0$
$x = -7$ or $x = -3$

Substitute each value of x into $y = 2x + 3$

When $x = -7$, $y = 2 \times -7 + 3 = -11$

When $x = -3$, $y = 2 \times -3 + 3 = -3$

Question 20

$x = -2, y = 7$ or $x = -7, y = 17$

Equate the right-handside of the two equations and solve the resulting quadratic equation.

$$\begin{aligned} x^2 + 7x + 17 &= -2x + 3 \\ x^2 + 9x + 14 &= 0 \\ (x+2)(x+7) &= 0 \\ x &= -2 \text{ or } x = -7 \end{aligned}$$

Substitute each value of x into $y = -2x + 3$

When $x = -2$, $y = -2 \times -2 + 3 = 7$

When $x = -7$, $y = -2 \times -7 + 3 = 17$

Question 21

$x = \frac{11}{17}, y = \frac{129}{17}$ or $x = -3, y = -7$

Substitute $y = 4x + 5$ into $x^2 + y^2 = 58$ then solve.

$$\begin{aligned} x^2 + (4x+5)^2 &= 58 \\ x^2 + (4x+5)(4x+5) &= 58 \\ x^2 + 16x^2 + 20x + 20x + 25 &= 58 \\ 17x^2 + 40x + 25 &= 58 \\ 17x^2 + 40x - 33 &= 0 \\ (17x - 11)(x+3) &= 0 \\ \therefore x = \tfrac{11}{17} \text{ or } x &= -3 \end{aligned}$$

Substitute these values into $y = 4x + 5$

$$\begin{aligned} \text{When } x = \tfrac{11}{17},\ y &= 4\left(\tfrac{11}{17}\right) + 5 \\ &= \tfrac{129}{17} \end{aligned}$$

$$\begin{aligned} \text{When } x = -3,\ y &= 4(-3) + 5 \\ &= -7 \end{aligned}$$

Question 22

$x = 5, y = 0$ or $x = 0, y = 5$

Rearrange $x + y = 5$ to make y the subject.

$$\begin{aligned} x + y &= 5 \\ y &= 5 - x \end{aligned}$$

Substitute $y = 5 - x$ into $x^2 + y^2 = 25$ then solve.

$$\begin{aligned} x^2 + (5-x)^2 &= 25 \\ x^2 + (5-x)(5-x) &= 25 \\ x^2 + 25 - 5x - 5x + x^2 &= 25 \\ 2x^2 - 10x + 25 &= 25 \\ 2x^2 - 10x &= 0 \\ \therefore\ x = 5 \text{ or } x &= 0 \end{aligned}$$

Substitute these values into $x + y = 5$

$$\begin{aligned} \text{When } x = 5,\ 1(5) + y &= 5 \\ y &= 0 \end{aligned}$$

$$\begin{aligned} \text{When } x = 0,\ 1(0) + y &= 5 \\ y &= 5 \end{aligned}$$

Question 23

$x = 5, y = 0$ or $x = 0, y = -5$

Rearrange $x - y = 5$ to make y the subject.

$$\begin{aligned} x - y &= 5 \\ -y &= 5 - x \\ y &= \frac{5-x}{-1} \\ y &= \frac{-5+x}{1} \end{aligned}$$

Substitute $y = \frac{-5+x}{1}$ into $x^2 + y^2 = 25$ then solve.

$$\begin{aligned} x^2 + \left(\frac{-5+x}{1}\right)^2 &= 25 \\ x^2 + \frac{(-5+x)(-5+x)}{1} &= 25 \\ x^2 + (-5+x)(-5+x) &= 25 \\ x^2 + 25 - 5x - 5x + x^2 &= 25 \\ 2x^2 - 10x + 25 &= 25 \\ 2x^2 - 10x &= 0 \\ \therefore\ x = 5 \text{ or } x &= 0 \end{aligned}$$

Substitute these values into $x - y = 5$

$$\begin{aligned} \text{When } x = 5,\ 1(5) - y &= 5 \\ y &= 0 \end{aligned}$$

$$\begin{aligned} \text{When } x = 0,\ 1(0) - y &= 5 \\ y &= -5 \end{aligned}$$

Question 24

$y = -3x^2 - 21x - 18$

&#9312Use the roots to set up the equation: $y = k(x + 6)(x + 1)$*Note: k is required as the 'leading coefficient' (a in the quadratic formula) might not be 1.*&#9313Substitute using a point on the curve:Since $(-2,12)$ is on the curve, we

$$12 = k(-2 + 6)(-2 + 1)$$

subsitute $x = -2$ and $y = 12$: $12 = k(-4)$ &#9314SimplifyUsing

$$k = -3$$

$$y = -3(x + 6)(x + 1)$$

$k = -3$, we get: $y = -3(x^2 + 7x + 6)$

$$y = -3x^2 - 21x - 18$$

Question 25

$$y = 5x^2 + 30x + 25$$

&#9312Use the roots to set up the equation: $y = k(x + 5)(x + 1)$*Note: k is required as the 'leading coefficient' (a in the quadratic formula) might not be 1.*&#9313Substitute using a point on the curve:Since $(-2,-15)$ is on the curve, we

$$-15 = k(-2 + 5)(-2 + 1)$$

subsitute $x = -2$ and $y = -15$: $-15 = k(-3)$ &#9314SimplifyUsing

$$k = 5$$

$$y = 5(x + 5)(x + 1)$$

$k = 5$, we get: $y = 5(x^2 + 6x + 5)$

$$y = 5x^2 + 30x + 25$$

Question 26

$$f(x) \geq -\frac{97}{12}$$

Sketch the graph of $y = f(x)$

Complete the square:

$$\begin{aligned} f(x) &= 3x^2 - 7x - 4 \\ &= 3\left(x^2 - \frac{7}{3}x\right) - 4 \\ &= 3\left(\left(x - \frac{7}{6}\right)^2 - \frac{49}{36}\right) - 4 \\ &= 3\left(x - \frac{7}{6}\right)^2 - \frac{97}{12} \end{aligned}$$

The minimum point of the graph has a y-coordinate of $-\frac{97}{12}$ $\therefore f(x) \geq -\frac{97}{12}$

Question 27

$\frac{35}{23}$

$$\begin{aligned} fg(10) &= f(g(10)) \\ &= f\left(\tfrac{5(10)-6}{4(10)-7}\right) \\ &= f\left(\tfrac{4}{3}\right) \\ &= f\left(\tfrac{10\left(\frac{4}{3}\right)+10}{4\left(\frac{4}{3}\right)+10}\right) \\ &= \tfrac{35}{23} \end{aligned}$$

Question 28

$100x + 110$

$$\begin{aligned} gg(x) &= g(g(x)) \\ &= g(\,10x + 10\,) \\ &= 10(10x + 10) + 10 \\ &= 100x + 110 \end{aligned}$$

Question 29

$\frac{x-5}{4}$

Replace $f(x)$ by y and make x the subject.

$$\begin{aligned} f(x) &= 4x + 5 \\ y &= 4x + 5 \\ y - 5 &= 4x \\ \tfrac{y-5}{4} &= x \end{aligned}$$

Interchange x and y and then replace y by $f^{-1}(x)$

$$\begin{aligned} \tfrac{x-5}{4} &= y \\ f^{-1}(x) &= \tfrac{x-5}{4} \end{aligned}$$

Question 30

$\frac{-x^2-7}{10x^2-6}$

① Write $y =$ ... and make x the subject.

$$\begin{aligned} y &= \sqrt{\frac{6x-7}{10x+1}} \\ y^2 &= \frac{6x-7}{10x+1} \\ 10xy^2+y^2 &= 6x-7 \\ 10xy^2-6x &= -y^2-7 \\ x\left(10y^2-6\right) &= -y^2-7 \\ x &= \frac{-y^2-7}{10y^2-6} \end{aligned}$$

② Interchange x and y and write $g^{-1}(x) = \ldots$

$$\begin{aligned} y &= \frac{-x^2-7}{10x^2-6} \\ g^{-1}(x) &= \frac{-x^2-7}{10x^2-6} \end{aligned}$$
